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What Works? Research into Practice

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Research Monograph # 1

How can teachers support meaningful, high-quality student interaction in the math classroom?

Student Interaction in the Math Classroom: Stealing Ideas or Building Understanding

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"It was the third math class of the year. My Grade 7 students were unusually eager. We were looking for patterns in a strategic list of solutions generated from a number game. As one student described a complex pattern in the sequence, a second student shouted: 'She stole my idea!' At that point, I knew my work was cut out for me. How could I possibly move this group of competitive students from believing that math was an individual sport where power lies in the hoarding of information and 'getting the answer first', to understanding the exponential power of mathematical thinking when it is shared and built collectively?"

Excerpted from a teacher's journal

Research tells us that student interaction – through classroom discussion and other forms of interactive participation – is foundational to deep understanding and related student achievement. But implementing discussion in the mathematics classroom has been found to be challenging.

The Value of Student Interaction

In the math reform literature, learning math is viewed as a social endeavour.[1,2] In this model, the math classroom functions as a community where thinking, talking, agreeing, and disagreeing are encouraged. The teacher provides students with powerful math problems to solve together and students are expected to justify and explain their solutions. The primary goal is to extend one's own thinking as well as that of others.[3]

Powerful problems are problems that allow for a range of solutions, or a range of problem-solving strategies. Math problems are powerful when they take students beyond the singular goal of computational mastery into more complex math thinking. Research has firmly established that higher-order questions are correlated with increased student achievement, particularly for conceptual understanding.[4] The benefits increase further when students share their reasoning with one another. Reform-based practices that emphasize student

Research Tells Us

- Teaching practices that emphasize student interaction improve both problem-solving and conceptual understanding – without loss of computational mastery.
- Benefits increase further when students share their reasoning with one another.
- Higher order questions are correlated with increased student achievement, particularly conceptual understanding.
- Left to their own devices, students will not necessarily engage in high-quality math-talk … the teacher plays an important role.

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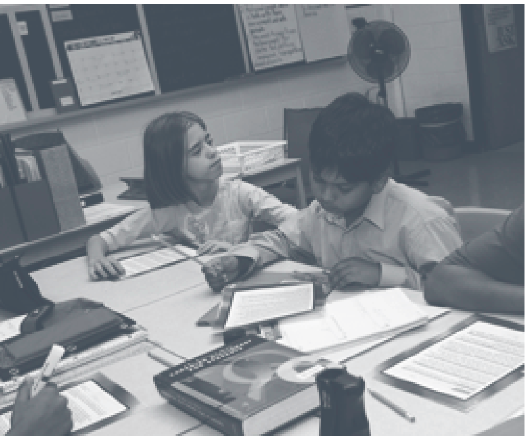

interaction improve both problem-solving and conceptual understanding[5,6] without the loss of computational mastery.[7,8] Why then does the traditional mathematics teaching model, focused on basic computational procedures with little facilitation of student discourse, continue to be the common instructional approach in many elementary schools?

Challenges that Teachers Face in Engaging Students

Math teachers face a number of challenges in facilitating high-quality student interaction, or "math-talk". The biggest is the complexity of trying to teach mathematics in ways they did not experience as students.[9,10] Discomfort for some with their own level of math content knowledge[11] and lack of sustained professional development opportunities also make teachers reluctant to adopt math-talk strategies.

Further, the complex negotiation of math-talk in the classroom requires facilitation skills and heightened attention to classroom dynamics. The teacher must model math-talk so that students understand the norms of interaction in the math classroom,[12] encourage students to justify their solutions and build on one another's ideas,[3] and finally step aside as students take increasing responsibility for sustaining and enriching interactions.

Time is another challenge. In the face of curricular demands, the time required for facilitated interaction has been identified by teachers as an inhibitor to implementing math-talk.[13] However, the research also tells us that despite these challenges, teachers have devised some particularly effective strategies for facilitating math-talk.

Challenges that Teachers Face

- complexities of teaching mathematics in ways they did not experience as students
- discomfort with their own mathematics knowledge
- lack of sustained professional development opportunities
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- lack of time, especially in face of curricular demands

The Teacher's Role

In an extensive study examining math classroom activity, student interaction was one of ten essential characteristics of effective mathematics teaching.[19] However, left to their own devices, students will not necessarily engage in high-quality math-talk. The teacher plays an important role. According to this same study, three main activities of Ontario teachers who successfully facilitated math-talk were :

1. The teacher assigned tasks that required students to work together to develop joint solutions and problem-solving strategies.
2. The teacher provided instruction on and modeled expected behaviours focusing on group skills, shared leadership, and effective math communication.
3. The teacher urged students to explain and compare their solutions and solution strategies with peers. Students were encouraged to be both supportive and challenging with peers.

Other research[15] has identified two more important roles:

4. The teacher knew when to intervene and when to let the conversation continue even if it was erroneous.
5. Students were evaluated on their math-talk.

Implications for Educational Practice

Five Strategies for Encouraging High-Quality Student Interaction

1. *The use of rich math tasks*

The quality of math tasks is of primary importance. When a task has multiple solutions and/or permits multiple solution strategies, students have increased opportunities to explain and justify their reasoning. If a task involves a simple

operation and single solution, there will be little or no opportunity to engage students.

2. *Justification of solutions*

Encouraging productive argumentation and justification in class discussions leads to greater student understanding. In a study of four teachers using the same lesson, Kazemi and Stipek[16] found that there were significant differences in the quality of math-talk from class to class. Two of the four classes demonstrated evidence of deeper mathematical inquiry. In these two classes, the teachers explicitly asked students to justify their strategies mathematically and not merely recount procedures.

3. *Students questioning one another*

Getting students to ask each other good questions is a very powerful strategy. For example, King[17] found that giving students prompt cards, with a range of higher-order questions, led to greater student achievement. The prompts were question stems such as "how are ... and ... similar?" Students applied current content to the questions (e.g., "how are squares and parallelograms similar?"). The students retained more when they used prompt cards than when they spent the same amount of time discussing content in small groups without prompts.

4. *Use of wait time*

Asking questions that call for higher-level thinking is not particularly helpful if students are not also given sufficient time to do the related thinking. Those teachers who increase the amount of time they give students to respond, allowing even three seconds instead of the usual one, have found that students give more detailed answers expressed with greater confidence. With increased wait time, combined with higher-level questions, student attitudes towards learning improve.[18]

5. *Use of guidelines for math-talk*

In a district-wide Grade 6 study, teachers were provided with professional development (PD) in mathematics content and pedagogical models for facilitating student interaction.[14] The results on EQAO mathematics assessments, in year-over-year comparisons before and after the PD opportunity, indicated a substantial increase in student achievement, while the reading and writing scores remained consistent. In this project, guidelines for whole-class math-talk were modeled with teachers in active PD sessions and were subsequently implemented by participating teachers. A year later, some teachers were observed using the guidelines, which were still posted in their classrooms. These guidelines (see sidebar) help teachers and students engage in high-quality interaction leading to richer mathematical thinking, and deeper understanding of concepts and related applications.

In sum ...

Let's return to the concern raised in the opening vignette, where shared or similar solutions and strategies are described as the "stealing" of ideas. In order to move beyond this competitive and isolating approach which has had limited success, students must be encouraged to work, think, and talk together while engaging in powerful mathematics tasks. Clearly, the teacher plays a pivotal role in shaping the learning environment. By providing students with a framework for interaction, students can be guided effectively towards working as a learning community in which sharing math power extends understanding and leads to higher levels of achievement.

Guidelines for Whole-Class Math-Talk

1. **Explain: "This is my solution/strategy ..." "I think _____ is saying that ..."**
   - Explain your thinking and show your thinking.
   - Rephrase what another student has said.
2. **Agree with reason: "I agree because ..."**
   - Agree with another student and describe your reason for agreeing.
   - Agree with another student and provide an alternate explanation.
3. **Disagree with reason: "I disagree because ..."**
   - Disagree with another student and explain or show how your thinking/solution differs.
4. **Build on: "I would like to build on that idea..."**
   - Build on the thinking of another student through explanation, example, or demonstration.
5. **Go beyond: "This makes me think about ..." "Another way to think about this is ..."**
   - Extend the ideas of other students by generalizing or linking the idea to another concept.
6. **Wait time:**
   - Wait to think about what is being said after someone speaks (try five seconds).

References

Sharing Best Practice

The Literacy and Numeracy Secretariat has developed a professional learning series to help classroom teachers enhance their mathematical knowledge and understanding:

- **Numeracy Professional Learning Series Regional workshops in January 2007**
- **Webcast on Mathematical Knowledge for Teaching with Dr. Deborah Loewenberg Ball www.curriculum.org**

The Ministry of Education has developed some new resources to share research on teaching and learning and on best practices in education, including:

- **Annual Ontario Education Research Symposium**
- ***Inspire: The Journal of Literacy and Numeracy for Ontario* www.inspirelearning.ca**
- ***Unlocking Potential for Learning: Effective District-Wide Strategies to Raise Student Achievement in Literacy and Numeracy***
- ***What Works? Research into Practice***

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